EN1800
Numerical Methods in engineering applications

Professor: Ronan Vicquelin

Language of instruction: English – **Number of hours**: 36 – **ECTS**: 3

Prerequisites: Calculus and vector analysis; Basics Python programming (a quick reminder is scheduled); Heat transfer; Fluid mechanics (appreciated but not necessary).

Period: S8 Elective 11 March to June IN28IE4, SEP8IE4

Course Objectives

Numerical simulations of physical phenomena have become inevitable. Indeed, on the one hand, benefiting from computational resources in the 21st century is common practice. On the other hand, due to the increasing complexity and transdisciplinary of practical engineering systems, no analytical solutions are available and the cost of experimental investigations becomes prohibitive. Therefore, engineers in charge of the design of such systems have no choice but to rely on numerical simulations.

The course objectives are:

. Understanding of standard numerical methods

. Applications of these methods in problem-solving workshop

. Critical analysis of simulations results

Combining their skills in computer science, heat transfer, fluid mechanics, mathematical analysis and numerical methods, the students will write their own programs from a blank page and answer a couple of practical engineering problems such as:

- Find the optimal residence time in a BioReactor.
- Identify the value and location of the maximum temperature of a heated structure.
- Risk of pollutant dispersion or flame flash-back.
- Prediction of recirculation length backwards a facing step
- …

On completion of the course, students should be able to

Spontaneously solve a simple problem with a small script to implement a numerical resolution; Formalize a physical problem into equations and identify their mathematical nature; Discretize a set of differential equations; Analyze the accuracy and stability of a numerical method; Derive an adapted numerical method in terms of accuracy and efficiency to solve the problem; Ensure the validity of the results though hypotheses checking and numerical errors characterization; Have a critical interpretation of the physical results; Solve problems found in engineering applications.

Course Contents

I. Basics on numerical approximations

· (1) **Introduction and Finite Differences.** The role of numerical methods in current design chains. Program, objectives, organization and methods. Introduction to numerical methods. Typical partial differential equations (PDE) found in engineering sciences. Classification of PDE. Finite differences. Approximations of derivatives using Taylor series. Order of the approximation. Centered and upwind formula. Taylor table to obtain general arbitrary order approximations.

· (2) **Numerical solution of ordinary differential equations**. Modified wave number of finite differences formula. Introduction to ordinary differential equations (ODE). Explicit Euler method.

Local and global order of accuracy. Stability of ODE numerical schemes. Implicit Euler and trapezoidal method. Linearization of implicit schemes. Stiffness.

None *II. Solving large linear equations systems: Applications to steady heat equation.*

· (3) **Elliptic PDE 1.** ODE: Runge-Kutta and multi-steps schemes. Examples of Elliptic PDE (steady diffusive problems). Boundary conditions. Direct vs iterative numerical methods. Jacobi and Gauss-Seidel methods.

· (4) **Elliptic PDE 2**. Succesive Over-Relaxation method. Conjuguate Gradient method. Notions on Krylov methods, preconditioning and multi-grid methods.

None *III. Methods for unsteady advection/diffusion problems*

· (5) **Hyperbolic and parabolic PDE: Explicit methods.** Examples (convection, unsteady diffusion processes). Application of explicit Euler method with finite differences. Stability analysis with semi-discrete form. Von Neumann stability analysis. CFL and Fourier criteria.

· (6) **Characterization of numerical errors**. Numerical dispersion and dissipation. Modified PDE after numerical discretization. Consistency. Interpretation of observed numerical instabilities. Fixes of centered advection: Lax-Friedrichs and Lax-Wendroff methods.

· (7) **Hyperbolic and parabolic PDE: Implicit methods.** Stiffness of unsteady heat equation. Resolution of 1D heat equation with implicit Euler method. Crank-Nicholson. Thomas algorithm. Multi-dimensional cases: Alternating Direction Implicit methods (ADI).

None *IV. Towards computational fluid dynamics*

· (8) **Methodology in numerical computations**. Problem definition and hypotheses. Non-dimensional equations. Mesh independent results. Post-processing. Hypotheses verifications.

· (9) **Incompressible Flow equations.** Governing equations of fluid motion (Euler and Navier-Stokes equations). Role of pressure in incompressible flows. Application of explicit Euler method. Poisson equation on pressure. Pressure/Velocity coupling.

· (10) **Semi-Implicit method for incompressible flows.** Predictor/Corrector steps to update the velocity field. Application of ADI method in the predictor step.

· (11) **Final project on incompressible flow.** Application of methodology rules. Compute forces.

Course Organization

Each lecture is followed by a problem-solving workshop on computers to (i) apply the concepts and methods seen in class and (ii) treat practical engineering applications in projects.

The whole course spans on 36 hours (33% theory, 67% practice).

Teaching Material and Textbooks

Lecture notes and problem notes.

Evaluation

Evaluation is based on the realization of four projects along the duration of the course: 2 small projects to hand out within a week, 2 comprehensive projects (mid-term and final). Deliverables are to be handed out as slides for each group of two people.