CentraleSupélec

casting — Devenez ingénieur centralien

June 2018

Written test - Mathematics

2 hours

The use of documents and calculators is forbidden.

The quality of writing is an important element for the evaluation.

Exercise 1

Q. 1.1 Justify the existence of

$$I = \int_{\mathbb{R}} e^{-x^2} \mathrm{d}x.$$

Q. 1.2 Compute the value of the integral I.

Exercise 2

Let P be a polynomial with real coefficients, $P \in \mathbb{R}[X]$. Assume all its roots are real. Let $\alpha \in \mathbb{R}$. We want to prove that all the roots of $P' + \alpha P$ are real too.

Q. 2.1 Assume at first that P has only simple roots. Show that $P' + \alpha P$, where P' the derivative polynomial of P, has only real simple roots, as well.

Q. 2.2 We do not assume that the roots of P are all simple anylonger. Show that the roots of $P' + \alpha P$ are all real.

Exercise 3

Q. 3.1 Determine the convergence radius R of the series $\sum \frac{n^n}{n!} z^n$.

Q. 3.2 Study the convergence of the series on the circle of center 0 and radius R.

Exercise 4

Q. 4.1 Study the linear independence of the following families:

1. $f_n : x \mapsto (\sin(x))^n$, $n \in \mathbb{N}$
2. $f_a : x \mapsto |x - a|$, $a \in \mathbb{R}$
3. $f_\alpha : x \mapsto \exp(\alpha x)$, $\alpha \in \mathbb{C}$
4. $(x \mapsto (\sin(x))^n), (x \mapsto (\cos(x))^n)$, $n \in \mathbb{N}$.

Q. 4.2 Let $n \geq 1$. Let $f_i : \mathbb{C} \to \mathbb{C}$, $i \in \{1, \dots, n\}$ be a family of functions. Set

$$\begin{aligned} \mathrm{A} : \mathbb{C}^n &\longrightarrow \mathscr{M}_\backslash(\mathbb{C}) \\ (x_1, \dots, x_n) &\longmapsto (f_i(x_j))_{1 \leq i, j \leq n} \end{aligned}$$

Find a necessary and sufficient condition on the function $\det(\mathrm{A})$ so that the family $(f_i)_{1 \leq i \leq n}$ is linearly independent.

Exercise 5

Let X be a real random variable following Poisson's law with parameter λ. Let Y be defined as

$$\mathrm{Y} = \begin{cases} \frac{\mathrm{X}}{2} & \text{if X is even} \\ 0 & \text{if X is odd} \end{cases}$$

Q. 5.1 Determine the law of Y.

Q. 5.2 Compute the expectation of Y.

Q. 5.3 Compute the variance of Y.